Tutorial 3 : CF and Symbolic Reachability

CS60030 Formal Systems

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FORMAL METHODS FOR SAFETY CRITICAL SYSTEMS

Characteristic Function : Gray Counter

a) Consider a 3-bit counter whose counting sequence is shown below.

000 $\rightarrow$ 001 $\rightarrow$ 011 $\rightarrow$ 010 $\rightarrow$ 110 $\rightarrow$ 111 $\rightarrow$ 101 $\rightarrow$ 100 $\rightarrow$ 000 ….

Here the state is represented by a vector $\langle x_1, x_2, x_3 \rangle$ of 3 state variables. Let $\langle x_1', x_2', x_3' \rangle$ denote the next state.

1) Develop the characteristic function, $cf(x_1, x_2, x_3, x_1', x_2', x_3')$, representing the transition relation of the counter.

Characteristic Function : Gray Counter

The equations for x_1', x_2', and x_3' can be written as

- $x_1' = \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3 + x_1 \bar{x}_2 x_3 = x_2 \bar{x}_3 + x_1 x_3$
- $x_2' = \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 \bar{x}_3 = x_2 \bar{x}_3 + \bar{x}_1 x_3$
- $x_3' = \bar{x}_1 \bar{x}_2 \bar{x}_3 + \bar{x}_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3 = x_1 x_2 + \bar{x}_1 \bar{x}_2$

Therefore, $cf = [x_1' \odot (x_2 \bar{x}_3 + x_1 x_3)][x_2' \odot (x_2 \bar{x}_3 + \bar{x}_1 x_3)][x_3' \odot (x_1 x_2 + \bar{x}_1 \bar{x}_2)]$

Symbolic Reachability : Gray Counter

b) We wish to determine whether the counter is a Gray counter. For this purpose we need to check from the transition relation of part (a) that successive states differ in only one bit. Prepare a Boolean formula, , such that the satisfiability of will enable you to determine whether the transition relation is one for the Gray counter?

Symbolic Reachability : Gray Counter

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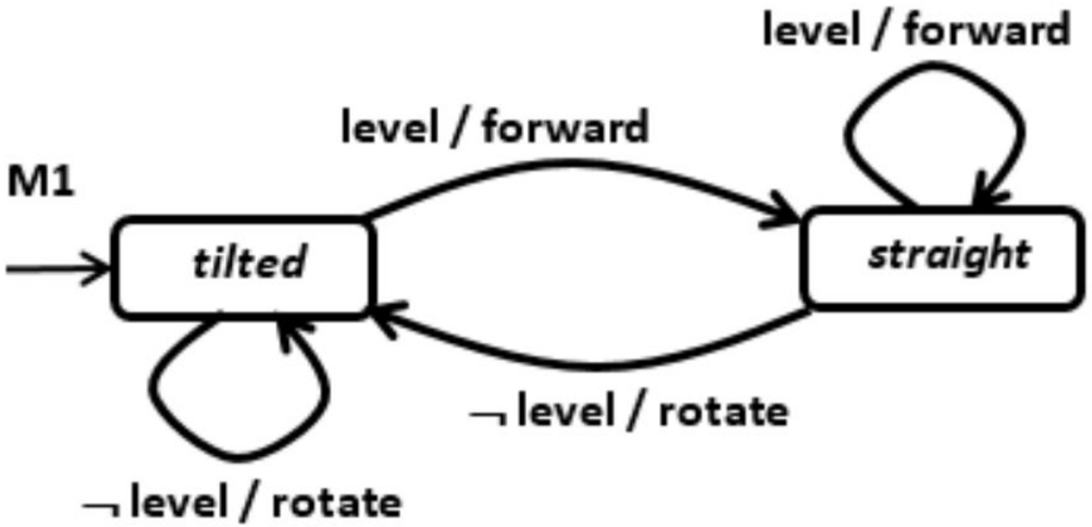
Symbolic Reachability : Gray Counter

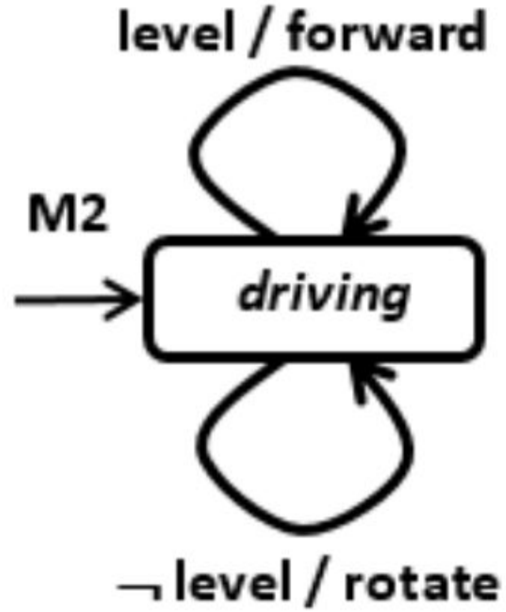
$$\psi = [(x_1' = \overline{x}_1) \leftrightarrow (x_2' = x_2)(x_3' = x_3)] \wedge$$
$$[(x_2' = \overline{x}_2) \leftrightarrow (x_1' = x_1)(x_3' = x_3)] \wedge$$
$$[(x_3' = \overline{x}_3) \leftrightarrow (x_1' = x_1)(x_2' = x_2)]$$

$$\therefore \phi = \neg\psi \wedge cf$$

Characteristics Function

Consider the finite state machines M1 and M2 for a robot. The variable, level, is an input to the machines, and the variables, forward and rotate, are outputs of the machines. Show the characteristic function representations of the transition relations of M1 and M2. You may use the first letter of each variable as a short form for convenience.

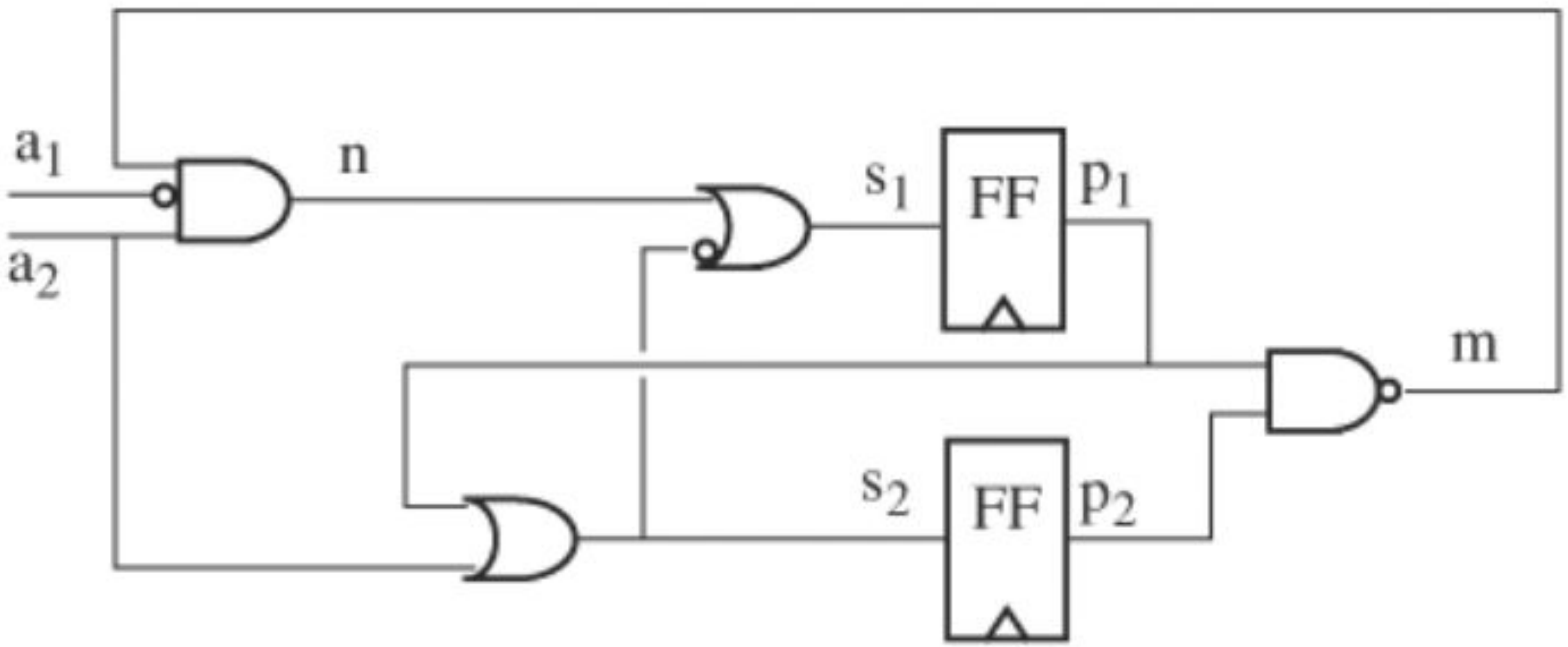




Two FSMs are called simulation equivalent or language equivalent if they produce a similar output sequence when executed with the same input sequence. Are M1 and M2 simulation equivalent?

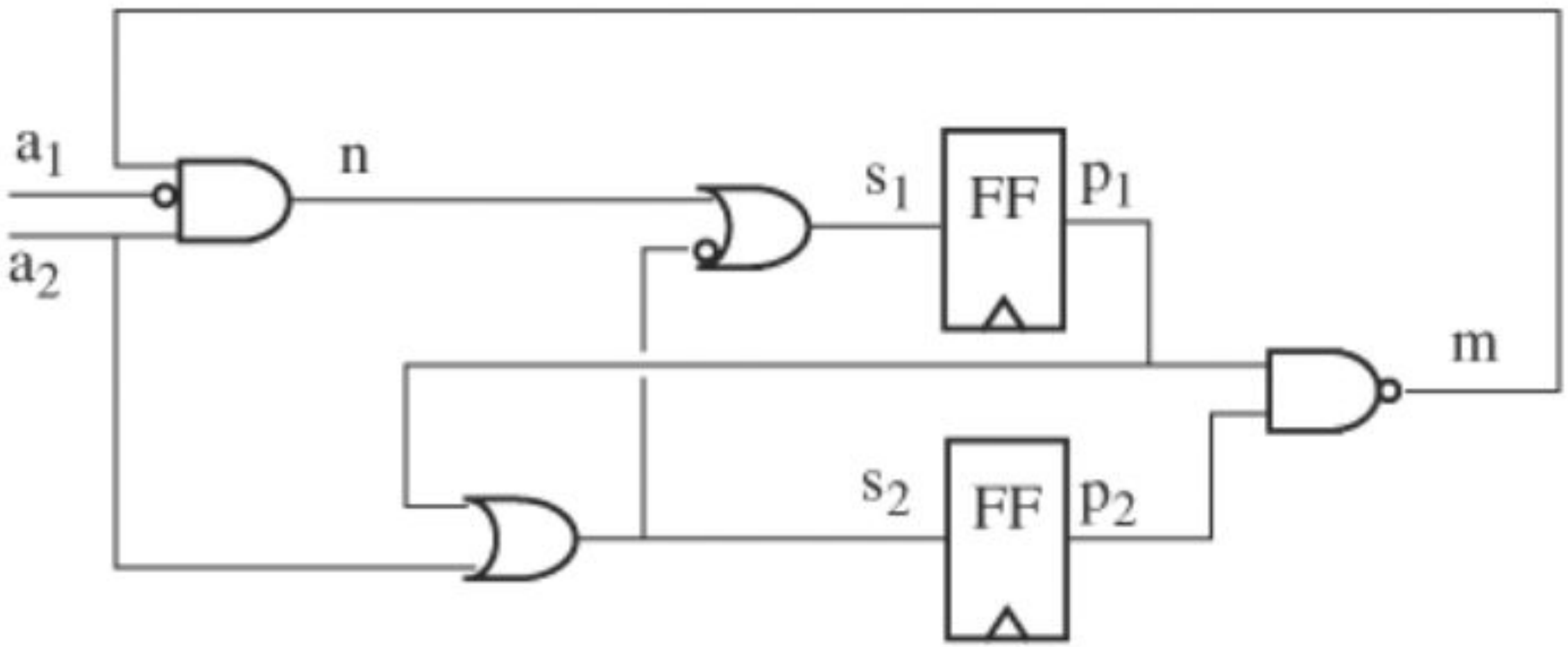
Symbolic Computation

2) Consider the following circuit. Formulate the encoding of s_1 and s_2 and represent the state transition function.



Symbolic Computation

Using the transition relation derived in part 1 encode the set of all next state for all possible inputs. What will be the set of all next states if the present state is either 01 or 10?



Symbolic Computation

s1 = s’2 + a’1a2 (p1p2)’

s2 = p1 + a2

$$T(p_1, p_2, s_1, s_2, a_1, a_2) = \overline{(s_1 \oplus (\overline{s_2} + \overline{a_1}a_2\overline{(p_1p_2)}))}\,\overline{(s_2 \oplus (p_1 + a_2))}$$

$$N(s_1, s_2) = \exists(a_1, a_2, p_1, p_2)T(p_1, p_2, s_1, s_2, a_1, a_2)P(p_1, p_2)$$

Symbolic Computation

Suppose TS = ⟨Q, R, Q_0⟩ is a transition system, where Q is the set of states, Q_0 is the set of initial states, R ⊆ Q X Q is the transition relation. The states in Q are defined by the state variables, ⟨ x_1, …, x_k ⟩, and therefore, the Boolean function, $R(x_1, \dots, x_k, x'_1, \dots, x'_k)$, defines the state transition relation in terms of the present state $\langle x_1, \dots, x_k \rangle$ and the next state $\langle x'_1, \dots, x'_k \rangle$. Write a Boolean function representing the states in Q that are reachable from Q_0 using two or three transitions, but not using a single transition.